INPUT IMPEDANCE CONDITIONS FOR MINIMIZING THE NOISE FIGURE OF AN ANALOG OPTICAL LINK

Edward Ackerman, Charles Cox III, Gary Betts, Harold Roussell, Kevin Ray, and Fred O'Donnell

Lincoln Laboratory, Massachusetts Institute of Technology
Lexington, MA 02173-9108

Abstract—It has been previously shown that 3 dB is the lowest noise figure attainable for an amplifierless optical link with perfect lossless impedance matching to the RF source. In a prior experimental link with near-perfect impedance matching, dissipative loss in our input matching circuit prevented us from achieving a measured noise figure of less than 4 dB. Investigation of the effects of input impedance mismatch indicates that mismatch can actually lower the noise figure to below 3 dB even in the presence of some dissipative loss in the input circuit. We have verified this theory by using the mismatch effect to reduce the measured noise figure of our link to 2.5 dB at 130 MHz. We believe this is the first demonstration of amplifierless link noise figure of less than 3 dB. We confirmed the validity of our measurement technique by also measuring the noise figure of a 2.5 dB RF attenuator to be 2.5 dB.

INTRODUCTION: PASSIVE MATCH LIMITS

Minimization of analog optical link noise figure is very important in applications such as remote sensing and receive antenna remoting. It is usually accomplished by using a low-noise amplifier before the modulation device (i.e., the external optical intensity modulator or directly modulated semiconductor laser). However, relying on large pre-amplifier gain to counteract a large link noise figure can yield a significantly smaller overall dynamic range. Therefore it is important to understand how the amplifierless link's noise figure can be minimized.

We previously reported [1] that when the modulation device in an intensity-modulation/direct-detection optical link is perfectly matched to the RF source impedance, the available gain and noise figure may be expressed, respectively, as follows:

$$G \equiv \frac{P_{out}}{P_{in,av}} = |g_1|^2 \frac{|Z_M|^2}{R_M} G_M N_D^2 R_{out}, \qquad (1)$$

$$NF = 10 \log\left[\frac{2}{G_M} + \frac{N_D^2 R_{out}}{kT\,G}\left(i_{RIN}^2 + i_{shot}^2\right) + \frac{1}{G}\right], \qquad (2)$$

where G_M is the input circuit's excess gain (less than 1 for a passive circuit), and where all other terms were defined in [1]. We named the $2/G_M$ term in equation (2) the general passive match limit to amplifierless link noise figure because it is the smallest noise figure attainable when perfect input impedance matching is achieved using passive components (i.e., when G_M is ≤ 1). Making the passive matching circuit lossless ($G_M = 1$) would yield the more familiar lossless passive match limit of $10 \times \log [2] = 3$ dB. Note from equation (2) that endeavoring to reach either the general or the lossless passive match limit requires a very large G, and that anything which reduces G causes NF to increase.

In a prior experiment we measured the NF of an amplifierless optical link that included a low-V_π external modulator we had matched to our 50 Ω RF source using a circuit for which G_M had been independently measured to be –0.7 dB. When we used a very large optical power (400 mW) at the input to the modulator we measured G=26.5 dB and NF=4.2 dB at 150 MHz. Using G=26.5 dB in equation (2) along with G_M = –0.7 dB and the other parameters in our link model resulted in a predicted noise figure of 4.0 dB. We interpreted these data as confirmation of the general passive match limit [1].

Since reporting this minimum link noise figure we have sought a means of decreasing it still further to below 3 dB. We have discovered that the lower limit to link noise figure is a strong function of the impedance mismatch between the RF source impedance and the link input impedance, and is not at its lowest when these two impedances are perfectly matched.

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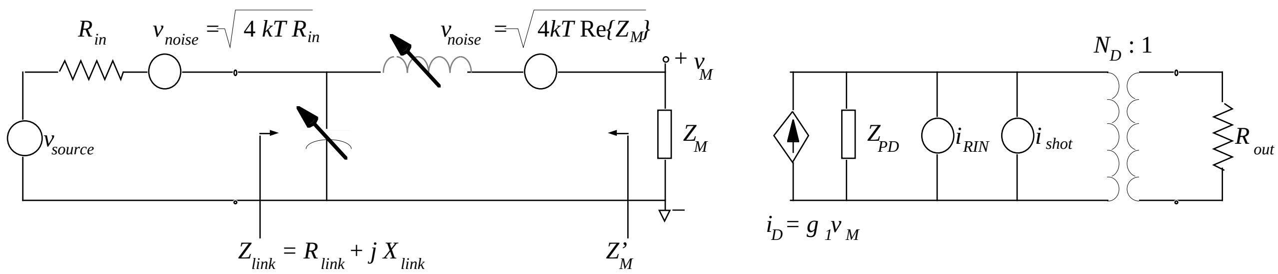

Figure 1. Equivalent circuit of amplifierless optical link with tunable interface circuit between the modulator and source impedances.

REMOVING THE "MATCH" CONSTRAINT

Figure 1 shows an equivalent circuit model of an amplifierless link with a tunable input impedance Z_{link} not necessarily matched to the source impedance R_{in}. The effects of input impedance mismatch on the amplifierless link's available gain and noise figure are evident from the following expressions for G and NF under a more general input impedance condition (we omit the derivations, which appear in [2]):

$$G = \frac{4\,R_{link}\,R_{in}}{\left|R_{link}+R_{in}\right|^2}\left|g_1\right|^2\frac{\left|Z_M\right|^2}{R_M}\,G_M\,N_D^2\,R_{out}, \tag{3}$$

$$NF = 10\ \log\left[1+\frac{1-G_M}{1+G_M}\frac{R_{link}}{R_{in}}+\frac{R_M^2\left(R_{link}+R_{in}\right)^2}{G_M\,R_{link}\,R_{in}\left|Z_M+Z'_M\right|^2}\right.$$
$$+\frac{1-G_M}{1+G_M}\frac{\left[\left(1+G_M\right)R_{in}+\left(1-G_M\right)R_{link}\right]^2}{4\,G_M\,R_{link}\,R_{in}}$$
$$\left.+\frac{N_D^2\,R_{out}}{kT\,G}\left(i_{RIN}^2+i_{shot}^2\right)+\frac{1}{G}\right], \tag{4}$$

where R_M is the real part of the modulation device's impedance, Z_M' is the impedance of the RF source as seen from the modulation device (through the interface circuit), and R_{link} is the real part of the link's input impedance. We derived G and NF for any complex link input impedance Z_{link}; however in equations (3) and (4) we show only the case where X_{link} (the imaginary part of Z_{link}) is equal to zero because without any loss of generality these equations allow us to illustrate our point more clearly than the more lengthy $X_{link} \neq 0$ equations. What equations (3) and (4) show is that so long as G is large even when $Z_{link} \neq R_M$, NF is not at its lowest when $Z_{link} = R_M$. This fact is even clearer when the lossless input circuit (i.e., $G_M=1$) case is examined:

$$G = \frac{4\,R_{link}\,R_{in}}{\left|R_{link}+R_{in}\right|^2}\left|g_1\right|^2\frac{\left|Z_M\right|^2}{R_M}\,N_D^2\,R_{out}\ , \tag{5}$$

$$NF = 10\ \log\left[1+\frac{R_M^2\left(R_{link}+R_{in}\right)^2}{R_{link}\,R_{in}\left|Z_M+Z'_M\right|^2}\right.$$
$$\left.+\frac{N_D^2\,R_{out}}{kT\,G}\left(i_{RIN}^2+i_{shot}^2\right)+\frac{1}{G}\right]. \tag{6}$$

Note that for very large G in combination with perfect impedance matching (i.e., when $R_{link}=R_{in}$ and $Z_M'=Z_M{}^*$), equation (6) reduces again to the lossless passive match limit of 3 dB. It appears from this expression that an effective way to obtain less than 3 dB noise figure is to design a modulator interface circuit to yield a link input impedance that causes the second term in equation (6) to be less than 1, and to do so using high-Q components in an effort to minimize the interface circuit loss. For this to be successful, the third and fourth terms in equation (6) must remain small even when mismatch causes G to be smaller than its maximum (i.e., perfect input impedance match) value.

EXPERIMENTAL RESULTS

Using the same link components described in [1], but this time with a modulator interface circuit having adjustable inductance and capacitance, we used the experimental set-up shown in Figure 2 to measure the link noise figure for many values of Z_{link}. As shown in the figure, we calibrated the HP 8970A noise figure meter with the low-ENR noise source before measuring the link noise figure. Varying the tunable reactance values in small increments, we were able to obtain a minimum noise figure of 2.5 dB at f=130 MHz, where we measured a corresponding value of Z_{link}=5+j5 Ω using a network analyzer. We then replaced the link under test with a variable RF attenuator, and adjusted the dial on this device until the meter measured its noise figure as 2.5 dB. At this dial setting we measured an insertion loss of –2.5 dB at f=130 MHz using the network analyzer. This verified that the meter had given an accurate noise figure measurement for the link.

To our knowledge 2.5 dB is the lowest noise figure ever reported for an amplifierless optical link. The significance of this result is not solely its record-breaking nature, but rather how it adds to our understanding of the relationship between link gain and noise figure under high-gain conditions. In effect what we have shown is that when a link has very large available gain it exhibits many of the same qualities as other high-gain devices such as BJTs and other transistors. Indeed, equations (5) and (6) above (or rather the more cumbersome versions of these that are valid for $X_{link}\neq 0$), which relate G and NF to the source and link input impedances, give rise to constant-G and -NF circles on a Smith Chart.

Using our model of the experimental link we have rendered such a Smith Chart plot in Figure 3(a). Note that what is plotted here is not Z_{link} but rather Z_M', the impedance presented to the modulator by the RF source (which lossless passive circuitry can transform to any impedance). Therefore the effects of matching circuit loss are ignored— just as they always are in similar Smith Chart plots published by transistor manufacturers, such as the one shown in Figure 3(b). Note on the link plot that the lossless passive match limit of 3 dB is represented by the point Z_M'=Z_M*, which is the perfect lossless match case. Due to the difficulty of measuring Z_M' after the input circuit was tuned for lowest NF, we currently are unable to project a point on this plot that correlates with our measured 2.5 dB noise figure; however it not likely be very close to a 2.5 dB circle, because this plot is valid only for lossless transformation of the source impedance R_{in} (50 Ω in our case) to Z_M'.

Two other facts about Figure 3 are worth noting. First, unlike the transistor, the link's unidirectionality (i.e., the fact that its S_{12} is exactly zero) causes the perfect match condition to always yield the highest gain, which is why the G=27.2 dB point in Figure 3(a) is at Z_M*. Second, if G for our link had not been so large, or had been less than 0 dB, the noise figure and gain contours would more nearly line up with each other; that is, the input matching condition yielding the highest G would also yield the lowest NF, as is the case for most links [and as equations (3) and (4) dictate].

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[3] "A low noise 4 GHz transistor amplifier using the HXTR-6101 silicon bipolar transistor," Hewlett-Packard Application Note 976, May 1975.

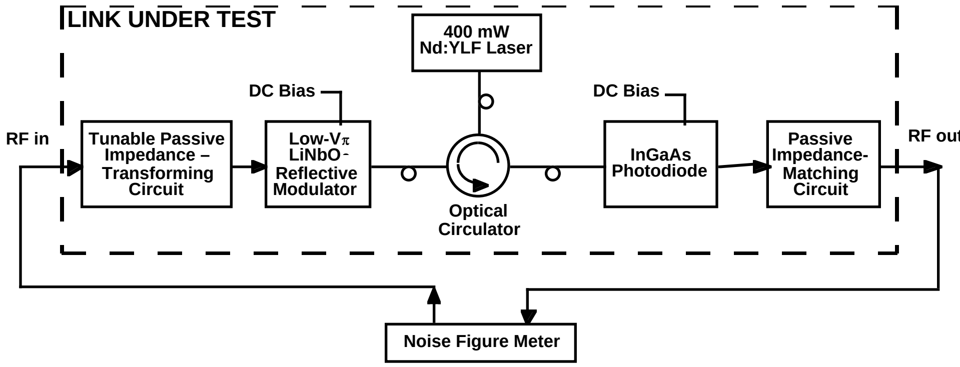

Figure 2. Set-up for measuring the noise figure of the experimental amplifierless link with tunable input impedance.

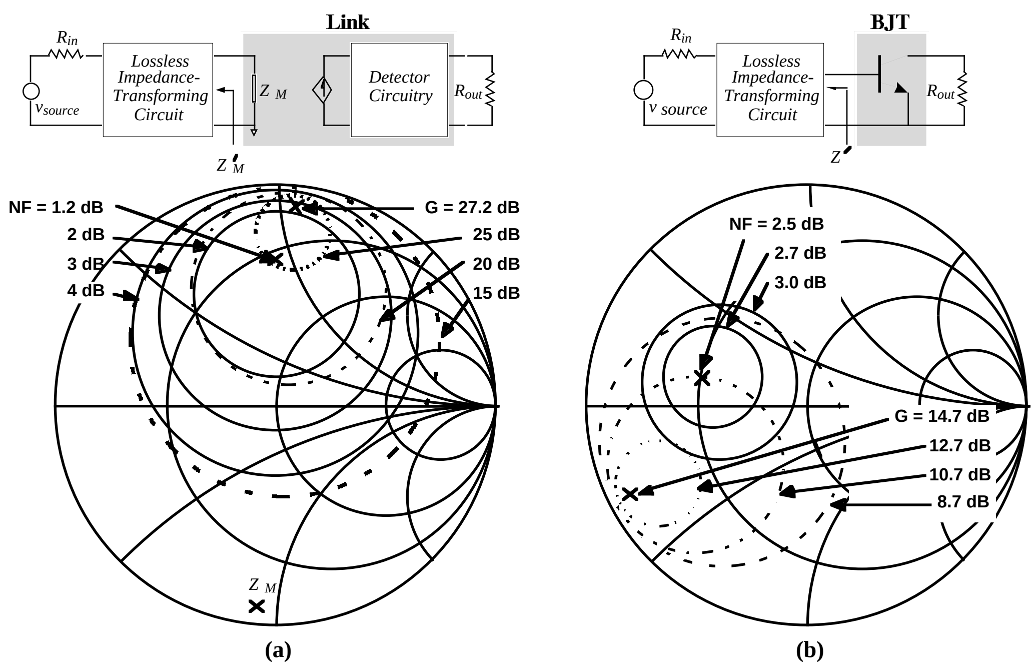

Figure 3. (a) Analytically-determined constant-*G* and constant-*NF* circles on a Smith Chart plot of the transformed source impedance Z_M' presented to the modulation device in the experimental amplifierless link.

(b) Analogous constant-*G* and constant-*NF* circles for an example bipolar transistor (from [3]).